

Censored Truncated Sequential Spectrum Sensing for Cognitive Radio Networks

Sina Maleki Geert Leus

Abstract

Reliable spectrum sensing is a key functionality of a cognitive radio network. Cooperative spectrum sensing improves the detection reliability of a cognitive radio system but also increases the system energy consumption which is a critical factor particularly for low-power wireless technologies. A censored truncated sequential spectrum sensing technique is considered as an energy-saving approach. To design the underlying sensing parameters, the maximum energy consumption per sensor is minimized subject to a lower bounded global probability of detection and an upper bounded false alarm rate. This way both the interference to the primary user due to miss detection and the network throughput as a result of a low false alarm rate is controlled. We compare the performance of the proposed scheme with a fixed sample size censoring scheme under different scenarios. It is shown that as the sensing cost of the cognitive radios increases, the energy efficiency of the censored truncated sequential approach grows significantly.

Index Terms

distributed spectrum sensing, sequential sensing, cognitive radio networks, censoring, energy efficiency.

I. INTRODUCTION

Wireless technologies have progressed rapidly during the recent years and have lead to a high demand for electromagnetic spectrum. The radio spectrum has been traditionally regularized for exploitation by licensed users, but this policy now results in spectrum scarcity [4]. Meanwhile, recent studies on spectrum utilization show that large parts of the licensed spectrum are highly underutilized in vast geographical locations and time periods [1], [2], [3]. Dynamic spectrum access based on cognitive radios has been proposed in order to opportunistically use these underutilized spectrum portions [4]. Regulatory bodies are currently working on the standardization, regulation, and modeling of such technologies with the goal of reaching a higher spectrum efficiency and availability for future wireless technologies [5], [6], [7], [8]. Our work is inspired by the recent FCC Report and Order permitting the operation of networks consisting of low-power devices and sensors in the VHF-UHF band [5] as well as by the IEEE 802.22 work group regulating the dynamic spectrum access for TV bands and wireless microphones [8].

S. Maleki and G. Leus are with the Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, 2628 CD Delft, The Netherlands (e-mail: s.maleki@tudelft.nl; g.j.t.leus@tudelft.nl). This work is supported in part by the NWO-STW under the VICI program (project 10382).

Recently, standardization of dynamic spectrum sharing of the 2.36-2.4 GHz band for body sensor networks has been initiated by the FCC [6] where all secondary users are consisting of low-power wireless devices.

Cognitive radios opportunistically share the spectrum while avoiding any harmful interference to the primary licensed users. They employ spectrum sensing to detect the empty portions of the radio spectrum, also known as spectrum holes. Upon detection of such a spectrum hole, cognitive radios dynamically share this hole. However, as soon as a primary user appears in the corresponding band, the cognitive radios have to vacate the band. This way, transmission is limited to the bands that are deemed to be empty in order to avoid interference to the primary users. As such, reliable spectrum sensing becomes a key functionality of a cognitive radio network.

The hidden terminal problem and fading effects have been shown to limit the reliability of spectrum sensing. Distributed cooperative detection has therefore been proposed to improve the detection performance of a cognitive radio network [9], [10], by exploiting spatial diversity among signal observations at spatially distributed sensors. However, such a gain in performance comes with a resulting higher network energy consumption which is a critical factor in a low-power radio system. Further, the maximum energy consumption of a low-power radio is limited by its battery. As a result, energy efficient spectrum sensing limiting the maximum energy consumption of a cognitive radio in a cooperative sensing framework is the focus of this paper.

Several distributed detection frameworks are discussed in [13], [14]. However, due to its simplicity and low delay, a parallel detection configuration is considered in the current paper where each secondary radio continuously senses the spectrum in periodic sensing slots. A local decision is then made at the radios and sent to the fusion center (FC), which makes a global decision about the presence (or absence) of the primary user and feeds it back to the cognitive radios. Several fusion schemes have been proposed in literature which can be categorized under soft and hard fusion strategies [13], [12]. In this paper, energy detection is employed for channel sensing which is a common approach to detect unknown signals [11], [12], and which leads to a comparable detection performance for hard and soft fusion schemes [10]. On the other hand, soft fusion requires several bits to be sent to the FC, while most of the hard fusion schemes require only one-bit transmissions. As a result, hard schemes are more energy efficient than soft schemes. From the above considerations, a hard fusion scheme is adopted in this paper. More specifically, the OR rule is employed at the FC due to its simple implementation.

A. Contributions

The spectrum sensing module consumes energy in both the sensing and transmission stage. A combination of censoring and truncated sequential sensing is proposed to save energy. The sensors sequentially sense the spectrum before reaching a truncation point where they are forced to stop sensing. If the accumulated energy of the collected sample observations is in a certain region before the truncation point, a decision is sent to the FC. Else, a censoring policy is used by the sensor, and no bits will be sent. This way, a large amount of energy is saved for both sensing and transmission. Our goal now is to minimize the maximum energy consumption per sensor subject to a specific detection performance constraint which is defined by a lower bound on the global probability of detection and an upper bound on the global probability of false alarm. In terms of cognitive radio system design, the probability of

detection limits the harmful interference to the primary user and the false alarm rate controls the loss in spectrum utilization. The ideal case yields no interference and full spectrum utilization, but it is practically impossible to reach this point. Hence, current standards determine a bound on the detection performance to achieve an acceptable interference and utilization level [8]. Analytical expressions for the underlying parameters are derived and it is shown that the problem can be solved by a two-dimensional search. However, to reduce the computational complexity of the system, a single-threshold truncated sequential test is proposed where each cognitive radio sends a decision to the FC upon the detection of the primary user. To make a fair comparison of the proposed technique with current energy efficient approaches, a fixed sample size censoring scheme is considered as a benchmark (it is simply called the censoring scheme throughout the rest of the paper) where each sensor employs a censoring policy after collecting a fixed number of samples. For this approach, it is proved that a single-threshold censoring policy is optimal in terms of energy consumption. Moreover, an explicit solution of the underlying problem is given.

B. Related work to censoring

Censoring has been thoroughly investigated in wireless sensor networks and cognitive radios [16]–[23]. It has been shown that censoring is very effective in terms of energy efficiency. In the early works, [18]–[21], the design of censoring parameters including lower and upper thresholds has been considered and mainly two problem formulations have been studied. In the Neyman-Pearson (NP) case, the miss-detection probability is minimized subject to a constraint on the probability of false alarm and average network energy consumption [19]–[21]. In the Bayesian case, on the other hand, the detection error probability is minimized subject to a constraint on the average network energy consumption. It is shown that when the constraint on the probability of false alarm is low enough (NP case) or the probability of target presence is much lower than the one for target absence (Bayesian case), a single-threshold censoring policy is optimal. Our fixed sample size censoring scheme is different from these works in several aspects. First, they have mainly considered a soft fusion scheme based on a likelihood ratio test (LRT) at the FC while in this paper a hard fusion OR rule is considered. Second, the optimization problem in our paper is different from the NP or Bayesian problems. Third, it is shown that in our scheme the optimal lower threshold is always zero and forth, an explicit solution of the underlying problem is given which has not yet been presented in the earlier works. A combination of censoring and sleeping is considered in [17] with the goal of maximizing the mutual information between the state of signal occupancy and the decision state of the FC, but the energy efficiency of the system is not directly addressed. Censoring for cognitive radios is considered in [16], [22], [23]. In [16], a censoring rule similar to the one in this paper is considered in order to limit the bandwidth occupancy of the cognitive radio network. Our fixed sample size censoring scheme is different in two ways. First, in [16], the FC makes no decision in case it does not receive any decision from the cognitive radios which is ambiguous, since the FC has to make a final decision, while in our paper, the FC reports the absence of a primary user, if no local decision is received at the FC. Second, we give a clear optimization problem and expression for the solution while this is not presented in [16]. In [22], analytical expressions for the sensing parameters are given according to an NP setup for both soft and hard fusion schemes, but unlike [18]–[21] no constraint on the energy consumption is

taken into account. As a result, our optimization problem is different than the one in [22]. A combined sleeping and censoring scheme is considered in [23] which can be viewed as the predecessor of this paper. The censoring scheme in this paper is different in some ways. First, in [23], the primary user signal is assumed deterministic, while in this paper the signal is assumed to be Gaussian. The optimization problem in the current paper is defined as the minimization of the maximum energy consumption per sensor while in [23], the total network energy consumption is minimized. For low-power radios, the problem in this paper makes more sense since the energy of individual radios is generally limited. Finally note that, the sleeping policy of [23] can be easily incorporated in our proposed censored truncated sequential sensing leading to even higher energy savings.

C. Related work to sequential sensing

Sequential detection as an approach to reduce the average number of sensors required to reach a decision is also studied comprehensively during the past decades [24]–[41]. In the context of distributed detection, the sensor observations are either spatially or temporally collected until the system comes up with a final decision [13], [32]. Intrinsic to every sequential sensing scheme, is a stopping rule and a terminal decision rule. The stopping rule is a function that determines when to stop collecting observations and therefore is a random variable. The terminal decision rule dictates which decision has to be made after the sequential test has stopped [32]. Since either the individual sensors or the FC can control the sequential test, two types of sequential detection can be recognized. When the FC manages the sequential test, [25], [27], [28], [31], [34], [37], [33], it either makes a decision or asks the sensors to send a new result. When the sequential test is carried out at the sensors, each sensor accumulates the samples sequentially and makes a decision about the presence or the absence of the target and then sends a binary decision to the FC [41], [35], [24], [29]. The other way to categorize sequential detection problems is based on the maximum number of samples that can be collected. In this context, we can distinguish between infinite horizon and finite horizon (or truncated) sequential detection [31] (the reader is referred to [13], [31] for a thorough analysis of distributed sequential detection). In [31], [30], each sensor collects a sequence of observations, constructs a summary message and passes it on to the FC and all other sensors. A Bayesian problem formulation comprising the minimization of the average error detection probability and sampling time cost over all admissible decision policies at the FC and all possible local decision functions at each sensor is then considered to determine the optimal stopping and decision rule. Further, algorithms to solve the optimization problem for both infinite and finite horizon are given. Our paper is different from [31] and [30] in the sense that we first consider a sequential detection scheme at each sensor and assume no communications among the sensors. Second, the optimization problem in this paper is an energy optimization problem and is constrained, while in [31], [30], the problem is different and is unconstrained. In [33], an infinite horizon sequential detection scheme based on the sequential probability ratio test (SPRT) at both the sensors and the FC is considered. Wald's analysis of error probability, [42], is employed to determine the thresholds at the sensors and the FC. Our sensing scheme is different, since we consider a truncated sequential detection and our thresholds are determined based on an energy optimization problem which do not lead to Wald's thresholds. The design of a distributed sequential detection network under a communication bandwidth

constraint is considered in [34]. Each sensor sends a quantized version of its observation to the FC and then the SPRT is employed to make the decision to stop or carry on sensing. The problem is formulated as to determine the distribution of the bandwidth among the sensors, the quantizer design, and the FC decision policy in order to minimize the average sample number (ASN). Incorporating [34] to increase the throughput of a cognitive radio system can be an interesting area of future research. [29] presents a distributed sequential sensing scheme where each sensor performs an SPRT and makes a decision. The decision is then sent to the FC and the FC announces the first incoming decision as the global decision. Henceforth, the global probability of detection and false alarm is equal to the ones at each sensor. This scheme can also be exploited to reduce the sensing and reporting time of the cognitive radio network thereby increasing the network throughput while decreasing the energy consumption. A combination of sequential detection and censoring is considered in [39]. Each sensor computes the LLR of the received sample and sends it to the FC, if it is deemed to be in a certain region. The FC then collects the received LLRs and as soon as their sum is larger than an upper threshold or smaller than a lower threshold, the decision is made and the sensors can stop sensing. The LLRs are sent in such a way that the larger LLRs are sent sooner. It is shown that the number of transmissions considerably reduces and particularly when the listening cost is high, this approach performs very well. However, our paper employs a hard fusion scheme at the FC, our sequential scheme is finite horizon, and further a clear optimization problem is given to optimize the energy consumption. [28] proposes a sequential censoring scheme where an SPRT is employed by the FC and soft or hard local decisions are sent to the FC according to a censoring policy. It is depicted that the number of transmissions decreases but on the other hand the ASN increases. Therefore, [28] ignores the effect of listening on the energy consumption and focuses only on the transmission energy which for current low-power radios is comparable to the sensing energy. In our work, we consider the energy of both sensing and transmission and optimize the overall energy consumed by each sensor. Further, since our sequential scheme is truncated, a decision will always be made by the FC, while in [28], the FC may not reach a decision in a reasonable time. Finally, the system in [28] asymptotically reaches a specific detection performance as the number of sensors grows, but this incurs a high total energy consumption by the system. As shall be shown later on, in our sequential censoring scheme, the energy consumption saturates when the number of cognitive radios increases. [35] considers a distributed sequential sensing scheme where each sensor employs the SPRT and upon reaching a decision, a binary result is sent to the FC. The FC then makes a final decision using a k-out-of-n rule. It is shown that for the same detection error probability, the detection performance of this sequential scheme is better than fixed size sampling and furthermore the observation energy is proven to be lower. The optimal sensing thresholds are found by an iterative algorithm that solves a Bayesian risk problem.

Sequential spectrum sensing is also considered for cognitive radio design. An infinite horizon SPRT is employed in [38], [37], [36], [27] for different sensing techniques. It is shown that the sensing time dramatically reduces when employing sequential detection. The optimization of cognitive network throughput under a constraint on the miss-detection probability is solved in [25], [26] in order to find the optimal stopping and access policies. This approach is infinite horizon which is not a valid assumption considering the limited sensing time of cognitive radios. Further, a binary result has to be sent to the FC for each collected observation sample which entails a high

transmission energy consumption. Nevertheless, the considered optimization problem is matched to the cognitive radio system requirements and an extension of [25] for the finite horizon case can also be considered. In [24], the sensing thresholds that minimize the ASN are derived subject to a constraint on the false alarm rate, miss-detection probability, outage probability and interference level. This method is particularly designed for systems with real-time traffic. A truncated sequential sensing technique is employed in [41] to reduce the sensing time of a cognitive radio system. The thresholds are determined such that a certain probability of false alarm and detection are obtained. In this paper, we are employing a similar technique, except that in [41], after the truncation point, a single threshold scheme is used to make a final decision, while in our paper, the sensor decision is censored if no decision is made before the truncation point. Here, a random signal is assumed for the primary user signal while in [41], the signal is assumed deterministic which leads to a different probability of detection and ASN. Further, [41] considers a single sensor detection scheme while we employ a distributed cooperative sensing system and finally, in our paper an explicit optimization problem is given to find the sensing parameters. Finally, clustered-based cognitive radio approaches have also been considered as a means to save energy (see e.g. [15]). Our approach can easily be implemented in a cluster-based system to gain more energy savings by a reduction in transmission energy.

The remainder of the paper is organized as follows. In Section II, the fixed size censoring scheme is described, including the optimization problem and the algorithm to solve it. The sequential censoring scheme is presented in Section III. Analytical expressions for the underlying system parameters are given and the optimization problem is analyzed. We discuss some numerical results in Section IV and conclusions and ideas for further work are posed in Section V.

II. FIXED SIZE CENSORING PROBLEM FORMULATION

A fixed size censoring scheme is discussed in this section as a benchmark for the main contribution of the paper in Section III, which studies a combination of sequential and censoring schemes. A network of M cognitive radios is considered under a cooperative spectrum sensing scheme. A parallel detection configuration is employed as shown in Fig. 1. Each cognitive radio senses the spectrum and makes a local decision about the presence or absence of the primary user and informs the FC by employing a censoring policy. The final decision is then made at the FC by employing the OR rule. The OR rule is used because of its simplicity and low implementation cost. Denoting r_{ij} to be the i -th sample received at the j -th cognitive radio, each radio solves a binary hypothesis testing problem as follows

$$\begin{aligned}\mathcal{H}_0 : r_{ij} &= w_{ij}, \quad i = 1, \dots, N, \quad j = 1, \dots, M \\ \mathcal{H}_1 : r_{ij} &= h_j s_{ij} + w_{ij}, \quad i = 1, \dots, N, \quad j = 1, \dots, M\end{aligned}\tag{1}$$

where w_{ij} is additive white Gaussian noise with zero mean and variance σ_w^2 , s_{ij} is the transmitted primary user signal which is also assumed to be white Gaussian with zero mean and variance σ_s^2 , and h_j is the channel gain between the primary user and the j -th cognitive radio which is assumed constant during each sensing period. Furthermore the s_{ij} 's and w_{ij} 's are assumed statistically independent.

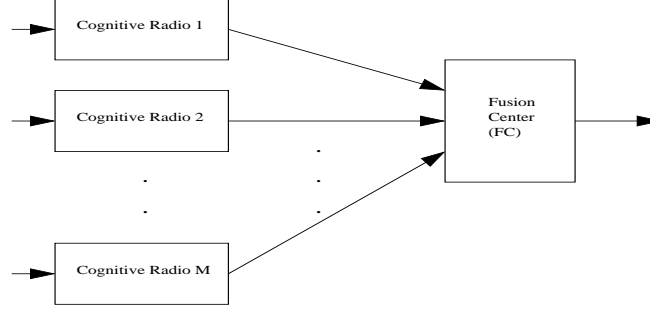


Fig. 1: Distributed spectrum sensing configuration

An energy detector is employed by each cognitive sensor which calculates the accumulated energy over N observation samples. The received energy collected over the N observation samples at the j -th radio is given by

$$\mathcal{E}_j = \sum_{i=1}^N \frac{|r_{ij}|^2}{\sigma_w^2}. \quad (2)$$

When the accumulated energy of the observation samples is calculated, a censoring policy is employed at each radio where the local decisions are sent to the FC only if they are deemed to be informative [23]. Censoring thresholds λ_1 and λ_2 are applied at each of the radios, where the range $\lambda_1 < \mathcal{E}_j < \lambda_2$ is called the censoring region. At the j -th radio, the local censoring decision rule is given by

$$\begin{cases} \text{send 1, declaring } \mathcal{H}_1 & \text{if } \mathcal{E}_j \geq \lambda_2, \\ \text{no decision} & \text{if } \lambda_1 < \mathcal{E}_j < \lambda_2, \\ \text{send 0, declaring } \mathcal{H}_0 & \text{if } \mathcal{E}_j \leq \lambda_1. \end{cases} \quad (3)$$

It is well known [12] that under such a model, \mathcal{E}_j follows a central chi-square distribution with $2M$ degrees of freedom under \mathcal{H}_0 and \mathcal{H}_1 and the related probability density functions are respectively given by

$$p(\mathcal{E}_j|\mathcal{H}_0) = \frac{1}{2^N \Gamma(N)} \mathcal{E}_j^{N-1} e^{-\mathcal{E}_j/2} I_{\{\mathcal{E}_j \geq 0\}}, \quad (4)$$

$$p(\mathcal{E}_j|\mathcal{H}_1) = \frac{1}{2^N \Gamma(N)} \mathcal{E}_j^{N-1} e^{-\mathcal{E}_j/2(1+\gamma_j)} I_{\{\mathcal{E}_j \geq 0\}}, \quad (5)$$

where, $I_{\{x \geq 0\}}$ is the indicator function, and $\gamma_j = |h_j|^2 \sigma_s^2 / \sigma_w^2$ is the SNR of the primary user received at the j -th cognitive radio. Based on the above decision rule, the local probabilities of false alarm and detection can be respectively written as

$$P_{fj} = Pr(\mathcal{E}_j \geq \lambda_2 | \mathcal{H}_0) = \frac{\Gamma(N, \frac{\lambda_2}{2})}{\Gamma(N)}, \quad (6)$$

$$P_{dj} = Pr(\mathcal{E}_j \geq \lambda_2 | \mathcal{H}_1) = \frac{\Gamma(N, \frac{\lambda_2}{2(1+\gamma_j)})}{\Gamma(N)}, \quad (7)$$

where $\Gamma(a, x)$ is the incomplete gamma function given by $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$, with $\Gamma(a, 0) = \Gamma(a)$.

Denoting C_{sj} and C_{ti} to be the energy consumed by the j -th radio in sensing per sample and transmission per bit, respectively, the average energy consumed for distributed sensing per user is given by,

$$C_j = NC_{sj} + (1 - \rho_j)C_{tj}, \quad (8)$$

where $\rho_j = Pr(\lambda_1 < \mathcal{E}_j < \lambda_2)$ is denoted to be the average censoring rate. Defining $\pi_0 = Pr(\mathcal{H}_0)$, $\pi_1 = Pr(\mathcal{H}_1)$, $\delta_{0j} = Pr(\lambda_1 < \mathcal{E}_j < \lambda_2 | \mathcal{H}_0)$ and $\delta_{1j} = Pr(\lambda_1 < \mathcal{E}_j < \lambda_2 | \mathcal{H}_1)$, ρ_j is given by

$$\begin{aligned}\rho_j &= Pr(\lambda_1 < \mathcal{E}_j < \lambda_2) \\ &= \pi_0 Pr(\lambda_1 < \mathcal{E}_j < \lambda_2 | \mathcal{H}_0) + \pi_1 Pr(\lambda_1 < \mathcal{E}_j < \lambda_2 | \mathcal{H}_1) \\ &= \pi_0 \delta_{0j} + \pi_1 \delta_{1j},\end{aligned}\tag{9}$$

with

$$\delta_{0j} = \frac{\Gamma(N, \frac{\lambda_1}{2})}{\Gamma(N)} - \frac{\Gamma(N, \frac{\lambda_2}{2})}{\Gamma(N)},\tag{10}$$

$$\delta_{1j} = \frac{\Gamma(N, \frac{\lambda_1}{2(1+\gamma_j)})}{\Gamma(N)} - \frac{\Gamma(N, \frac{\lambda_2}{2(1+\gamma_j)})}{\Gamma(N)}.\tag{11}$$

Denoting Q_F^c and Q_D^c to be the respective global probability of false alarm and detection, the target detection performance is then quantified by $Q_F^c \leq \alpha$ and $Q_D^c \geq \beta$, where, α and β are pre-specified detection design parameters. In practice, it is desirable to have α close to zero and β close to unity, which respectively ensure that the cognitive sensor network can exploit empty channels and that primary users are not interfered with. Our goal is to determine the optimum censoring thresholds λ_1 and λ_2 such that the maximum energy consumption per sensor, i.e., $\max_j C_j$, is minimized subject to the constraints $Q_F^c \leq \alpha$ and $Q_D^c \geq \beta$. Hence, our optimization problem can be formulated as

$$\min_{\lambda_1, \lambda_2} \max_j C_j\tag{12}$$

$$\text{s.t. } Q_F^c \leq \alpha, \quad Q_D^c \geq \beta.\tag{13}$$

The FC employs an OR rule to make the final decision which is denoted by D_{FC} , i.e., $D_{FC} = 1$ if the FC receives at least one local decision declaring 1, else $D_{FC} = 0$. This way, the global probability of false alarm and detection can be derived as

$$Q_F^c = Pr(D_{FC} = 1 | \mathcal{H}_0) = 1 - \prod_{j=1}^M (1 - P_{fj}),\tag{14}$$

$$Q_D^c = Pr(D_{FC} = 1 | \mathcal{H}_1) = 1 - \prod_{j=1}^M (1 - P_{dj}).\tag{15}$$

Note that since all the cognitive radios employ the same upper threshold λ_2 , we can state that $P_{fj} = P_f$ defined in (6). As a result, (14) becomes

$$Q_F^c = 1 - (1 - P_f)^M.\tag{16}$$

Theorem 1. The optimal solution of (12) is obtained by $\lambda_1 = 0$.

Proof. Since $\frac{\partial C_j}{\partial \rho_j} = -C_{tj} \leq 0$ and $\frac{\partial \rho_j}{\partial \lambda_1} = -\frac{\pi_0}{2^N \Gamma(N)} \lambda_1^{N-1} e^{\lambda_1/2} I_{\{\lambda_1 \geq 0\}} - \frac{\pi_1}{2^N \Gamma(N)} \lambda_1^{N-1} e^{\lambda_1/2(1+\gamma_j)} I_{\{\lambda_1 \geq 0\}} \leq 0$, we obtain $\frac{\partial C_j}{\partial \lambda_1} = \frac{\partial C_j}{\partial \rho_j} \frac{\partial \rho_j}{\partial \lambda_1} \geq 0$, and thus the optimal C_j is attained for the lowest λ_1 in the feasible set of the problem which is equal to 0. \square

From Theorem 1, (10) and (11) can be simplified to $\delta_{0j} = 1 - P_f$ and $\delta_{1j} = 1 - P_{dj}$ and so (12) becomes,

$$\min_{\lambda_2} \max_j [NC_{sj} + (\pi_0 P_f + \pi_1 P_{dj})C_{tj}] \quad (17)$$

$$\text{s.t. } 1 - (1 - P_f)^M \leq \alpha, \quad 1 - \prod_{j=1}^M (1 - P_{dj}) \geq \beta. \quad (18)$$

Since there is a one-to-one relationship between λ_2 and P_f , i.e., $\lambda_2 = 2\Gamma^{-1}(N, \Gamma(N)P_f)$, (17) can be formulated as [43, p.130],

$$\min_{P_f} \max_j [NC_{sj} + (\pi_0 P_f + \pi_1 P_{dj})C_{tj}] \quad (19)$$

$$\text{s.t. } 1 - (1 - P_f)^M \leq \alpha, \quad 1 - \prod_{j=1}^M (1 - P_{dj}) \geq \beta.$$

Defining $P_f = F(\lambda_2) = \frac{\Gamma(N, \frac{\lambda_2}{2})}{\Gamma(N)}$ and $P_{dj} = G_j(\lambda_2) = \frac{\Gamma(N, \frac{\lambda_2}{2(1+\gamma_j)})}{\Gamma(N)}$, we can write P_{dj} as $P_{dj} = G_j(F^{-1}(P_f))$.

Calculating the derivative of C_j with respect to P_f , we find that

$$\frac{\partial C_j}{\partial P_f} = \frac{\partial [C_{tj}(\pi_0 P_f + \pi_1 P_{dj})]}{\partial P_f} = C_{tj}\pi_0 + \frac{\partial P_{dj}}{\partial P_f} \geq 0, \quad (20)$$

where we use the fact that

$$\frac{\partial P_{dj}}{\partial P_f} = \frac{-\frac{1}{2^N \Gamma(N)} 2\Gamma^{-1}[N, \Gamma(N)P_f]^{N-1} e^{2\Gamma^{-1}[N, \Gamma(N)P_f]/2(1+\gamma_j)} I_{\{2\Gamma^{-1}[N, \Gamma(N)P_f] \geq 0\}}}{-\frac{1}{2^N \Gamma(N)} 2\Gamma^{-1}[N, \Gamma(N)P_f]^{N-1} e^{2\Gamma^{-1}[N, \Gamma(N)P_f]/2} I_{\{2\Gamma^{-1}[N, \Gamma(N)P_f] \geq 0\}}} \geq 0. \quad (21)$$

Therefore, we can simplify (19) as

$$\min_{P_f} P_f \quad (22)$$

$$\text{s.t. } 1 - (1 - P_f)^M \leq \alpha, \quad 1 - \prod_{j=1}^M (1 - P_{dj}) \geq \beta.$$

which can be easily solved by a line search over P_f . However, since Q_D^c is a monotonically increasing function of P_f , i.e., $Q_D^c = H(P_f) = 1 - \prod_{j=1}^M (1 - G_j(F^{-1}(P_f)))$ and so $\frac{\partial Q_D^c}{\partial P_f} = \frac{\partial Q_D^c}{\partial P_{dj}} \frac{\partial P_{dj}}{\partial P_f} = \prod_{l=1, l \neq j}^M (1 - P_{dl}) \frac{\partial P_{dj}}{\partial P_f} \geq 0$, we can further simplify the constraints in (22) as $P_f \leq 1 - (1 - \alpha)^{1/M}$ and $P_f \geq H^{-1}(\beta)$. Thus, we obtain

$$\min_{P_f} P_f \quad (23)$$

$$\text{s.t. } P_f \leq 1 - (1 - \alpha)^{1/M}, \quad P_f \geq H^{-1}(\beta).$$

Therefore, if the feasible set of (23) is not empty, then the optimal solution is given by $P_f = H^{-1}(\beta)$. When the received SNR of the primary user by the cognitive radios can be assumed to be the same, the local probabilities of detection will be all the same, i.e., $P_{dj} = P_d = G(F^{-1}(P_f))$, and thus $Q_D^c = 1 - (1 - P_d)^M = 1 - (1 - G(F^{-1}(P_f)))^M$. This way the optimal P_d is $P_d = 1 - (1 - \beta)^{1/M}$ and the optimal P_f is given by $P_f = F(G^{-1}(1 - (1 - \beta)^{1/M}))$. Note that such an assumption is considered a good assumption if the difference between the SNRs is less than 1 dB which holds in many practical situations [44]. Furthermore, by increasing the SNR, the optimal P_f decreases and so does the maximum energy consumption per sensor. Therefore, one suboptimal solution for (23) is to assume that the SNR for all the cognitive radios is equal to the minimum SNR and to find the sensing parameters using the earlier mentioned P_f and P_d . This way we are certain that the probability of detection constraint is satisfied because $\beta \leq Q_D^c(\gamma_{\min} = \min\{\gamma_1, \dots, \gamma_M\}) \leq Q_D^c(\gamma_1, \dots, \gamma_M)$. Although, the censoring scheme gives a considerable energy saving as shown in Section IV, it only relies on the transmission cost minimization. In the

following section, a combination of censoring and sequential approaches is presented which optimizes both the sensing and the transmission cost.

III. SEQUENTIAL CENSORING PROBLEM FORMULATION

A. System Model

Unlike Section II, where each user collects a specific number of samples, in this section, each cognitive radio sequentially senses the spectrum and upon reaching a decision about the presence or absence of the primary user, it sends the result to the FC by employing a censoring policy as introduced in Section II. The final decision is then made at the FC by employing the OR rule. Here, a censored truncated sequential sensing scheme is employed where each cognitive radio carries on sensing until it reaches a decision while not passing a limit of N samples. Denoting Λ_{nj} to be the decision statistic at the j -th cognitive radio after n consecutive samples, the local decision rule to make a final decision is as follows,

$$\begin{cases} \text{send 1, declaring } \mathcal{H}_1 & \text{if } \Lambda_{nj} \geq b \text{ and } n \in [1, N], \\ \text{continue sensing} & \text{if } \Lambda_{nj} \in (a, b) \text{ and } n \in [1, N], \\ \text{no decision} & \text{if } a < \Lambda_{nj} < b \text{ and } n = N, \\ \text{send 0, declaring } \mathcal{H}_0 & \text{if } \Lambda_{nj} \leq a \text{ and } n \in [1, N], \end{cases} \quad (24)$$

where $a < 0$ and $b > 0$. To avoid the calculation of the LLR for each sample and because of the simple implementation of an energy detector, a sequential shifted chi-square test is employed, as proposed in [41]. Therefore, the decision metric Λ_{nj} is defined as follows

$$\Lambda_{nj} = \sum_{i=1}^n (|r_{ij}|^2 - \Lambda), \quad (25)$$

where $\sigma_w^2 < \Lambda < \sigma_w^2(1 + \gamma_j)$ is a predetermined constant and $\gamma_j = |h_j|^2 \sigma_s^2 / \sigma_w^2$ is the SNR of the primary user received at the j -th cognitive radio. Dividing left and right hand sides of (25) by σ_w^2 we obtain

$$\bar{\Lambda}_{nj} = \Lambda_{nj} / \sigma_w^2 = \sum_{i=1}^n (|r_{ij}|^2 - \Lambda) / \sigma_w^2. \quad (26)$$

The probability density function of $x_{ij} = |r_{ij}|^2 / \sigma_w^2$ under \mathcal{H}_0 and \mathcal{H}_1 is a chi-square distribution with $2n$ degrees of freedom. Thus, x_{ij} becomes exponentially distributed under both \mathcal{H}_0 and \mathcal{H}_1 . Henceforth, we obtain

$$Pr(x_{ij} | \mathcal{H}_0) = \frac{1}{2} e^{-x_{ij}/2} I_{\{x_{ij} \geq 0\}}, \quad (27)$$

$$Pr(x_{ij} | \mathcal{H}_1) = \frac{1}{2(1 + \gamma_j)} e^{-x_{ij}/2(1 + \gamma_j)} I_{\{x_{ij} \geq 0\}}, \quad (28)$$

Defining $\zeta_{nj} = \sum_{i=1}^n |r_{ij}|^2 / \sigma_w^2 = \sum_{i=1}^n x_{ij}$, it is clear that, $\bar{\Lambda}_{nj} = \zeta_{nj} - n\bar{\Lambda}$, where $\bar{\Lambda} = \Lambda / \sigma_w^2$. Denoting $a_i = 0$, $i = 1, \dots, p$, $a_i = \bar{a} + i\bar{\Lambda}$, $i = p + 1, \dots, N$ and $b_i = \bar{b} + i\bar{\Lambda}$, $i = 1, \dots, N$, where $\bar{a} = a / \sigma_w^2$ and $\bar{b} = b / \sigma_w^2$,

and where $p = \lfloor -a/\Lambda \rfloor$, (24) becomes

$$\begin{cases} \text{send 1, declaring } \mathcal{H}_1 & \text{if } \zeta_{nj} \geq b_n \text{ and } n \in [1, N], \\ \text{continue sensing} & \text{if } \zeta_{nj} \in (a_n, b_n) \text{ and } n \in [1, N], \\ \text{no decision} & \text{if } \zeta_{nj} \in (a_n, b_n) \text{ and } n = N, \\ \text{send 0, declaring } \mathcal{H}_0 & \text{if } \zeta_{nj} \leq a_n \text{ and } n \in [1, N]. \end{cases} \quad (29)$$

Defining $\zeta_{0j} = 0$, the local probability of false alarm at the j -th cognitive radio, P_{fj} , can be written as

$$P_{fj} = \sum_{n=1}^N Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{n-1j} \in (a_{n-1}, b_{n-1}), \zeta_{nj} \geq b_n | \mathcal{H}_0), \quad (30)$$

whereas the local probability of detection, P_{dj} , is obtained as follows

$$P_{dj} = \sum_{n=1}^N Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{n-1j} \in (a_{n-1}, b_{n-1}), \zeta_{nj} \geq b_n | \mathcal{H}_1). \quad (31)$$

Denoting ρ_j to be the average censoring rate at the j -th cognitive radio, and δ_{0j} and δ_{1j} to be the respective average censoring rate under \mathcal{H}_0 and \mathcal{H}_1 , we have

$$\begin{aligned} \rho_j &= Pr(\zeta_{1j} \in (a_1, b_1), \dots, \zeta_{Nj} \in (a_N, b_N)) \\ &= \pi_0 Pr(\zeta_{1j} \in (a_1, b_1), \dots, \zeta_{Nj} \in (a_N, b_N) | H_0) \\ &\quad + \pi_1 Pr(\zeta_{1j} \in (a_1, b_1), \dots, \zeta_{Nj} \in (a_N, b_N) | H_1) \\ &= \pi_0 \delta_{0j} + \pi_1 \delta_{1j}, \end{aligned} \quad (32)$$

where

$$\delta_{0j} = Pr(\zeta_{1j} \in (a_1, b_1), \dots, \zeta_{Nj} \in (a_N, b_N) | H_0), \quad (33)$$

$$\delta_{1j} = Pr(\zeta_{1j} \in (a_1, b_1), \dots, \zeta_{Nj} \in (a_N, b_N) | H_1). \quad (34)$$

The other parameter that is important in any sequential detection scheme is the average sample number (ASN) required to reach a decision. Denoting N_j to be a random variable representing the number of samples required to announce presence or absence of the primary user, the ASN for the j -th cognitive radio, denoted as $\bar{N}_j = E(N_j)$, can be defined as

$$\bar{N}_j = \pi_0 E(N_j | \mathcal{H}_0) + \pi_1 E(N_j | \mathcal{H}_1), \quad (35)$$

where

$$\begin{aligned} E(N_j | \mathcal{H}_0) &= \sum_{n=1}^N n Pr(N_j = n | \mathcal{H}_0) \\ &= \sum_{n=1}^{N-1} n [Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{n-1j} \in (a_{n-1}, b_{n-1}) | \mathcal{H}_0) \\ &\quad - Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{nj} \in (a_n, b_n) | \mathcal{H}_0)] \\ &\quad + N Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{N-1j} \in (a_{N-1}, b_{N-1}) | \mathcal{H}_0), \end{aligned} \quad (36)$$

and

$$\begin{aligned}
E(N_j|\mathcal{H}_1) &= \sum_{n=1}^N nPr(N_j = n|\mathcal{H}_1) \\
&= \sum_{n=1}^{N-1} n[Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{nj} \in (a_{n-1}, b_{n-1})|\mathcal{H}_1) \\
&\quad - Pr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{nj} \in (a_n, b_n)|\mathcal{H}_1)] \\
&\quad + NPr(\zeta_{0j} \in (a_0, b_0), \dots, \zeta_{N-1j} \in (a_{N-1}, b_{N-1})|\mathcal{H}_1).
\end{aligned} \tag{37}$$

Denoting C_{sj} to be the sensing energy of one sample and C_{tj} to be the transmission energy of a decision bit at the j -th cognitive radio, the total average energy consumption at the j -th cognitive radio becomes

$$C_j = \bar{N}_j C_{sj} + (1 - \rho_j) C_{tj}. \tag{38}$$

Denoting Q_F^{cs} and Q_D^{cs} to be the respective global probabilities of false alarm and detection, we define our problem as the minimization of the maximum energy consumption over all cognitive radios subject to a constraint on the global probabilities of false alarm and detection as follows

$$\begin{aligned}
&\min_{\bar{a}, \bar{b}} \max_j C_j \\
&\text{s.t. } Q_F^{cs} \leq \alpha, \quad Q_D^{cs} \geq \beta.
\end{aligned} \tag{39}$$

As in (14), the global probability of false alarm is

$$Q_F^{cs} = Pr(D_{FC} = 1|\mathcal{H}_0) = 1 - \prod_{j=1}^M (1 - P_{fj}), \tag{40}$$

and the global probability of detection is

$$Q_D^{cs} = Pr(D_{FC} = 1|\mathcal{H}_1) = 1 - \prod_{j=1}^M (1 - P_{dj}). \tag{41}$$

Note that since $P_{f1} = \dots = P_{fM}$, in the rest of the paper, it is assumed that $P_{fj} = P_f$.

In the following subsection, analytical expressions for the probability of false alarm and detection as well as the censoring rate and ASN are extracted.

B. Parameter and Problem Analysis

Looking at (30), (31), (32) and (35), we can see that the joint probability distribution function of $p(\zeta_{1j}, \dots, \zeta_{nj})$ is the foundation of all the equations. Since, $x_{ij} = \zeta_{ij} - \zeta_{i-1j}$ for $i = 1, \dots, N$, we have,

$$\begin{aligned}
 p(\zeta_{1j}, \dots, \zeta_{nj}) &= p(\zeta_{2j}, \dots, \zeta_{nj} | \zeta_{1j}) p(\zeta_{1j}) \\
 &= p(\zeta_{3j}, \dots, \zeta_{nj} | \zeta_{1j}, \zeta_{2j}) p(\zeta_{2j} | \zeta_{1j}) p(\zeta_{1j}) \\
 &= . \\
 &= . \\
 &= p(\zeta_{nj} | \zeta_{1j}, \dots, \zeta_{n-1j}) \dots p(\zeta_{1j}) \\
 &= p(x_{nj}) p(x_{n-1j}) \dots p(x_{1j}).
 \end{aligned} \tag{42}$$

Therefore, the joint probability distribution function under \mathcal{H}_0 and \mathcal{H}_1 becomes

$$p(\zeta_{1j}, \dots, \zeta_{nj} | \mathcal{H}_0) = \frac{1}{2^n} e^{-\zeta_{nj}/2} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{nj}\}}, \tag{43}$$

$$p(\zeta_{1j}, \dots, \zeta_{nj} | \mathcal{H}_1) = \frac{1}{[2(1 + \gamma_j)]^n} e^{-\zeta_{nj}/2(1 + \gamma_j)} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{nj}\}}, \tag{44}$$

where, $I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{nj}\}}$ is again the indicator function.

The local probability of false alarm and the ASN under \mathcal{H}_0 in this work are similar to the one that is considered in [40] and [41]. The difference is that in [41], if the cognitive radio does not reach a decision after N samples, it employs a single threshold decision policy to give a final decision about the presence or absence of the cognitive radio. Henceforth, with a small modification we can use the results in [41] for our analysis. Further, since in our work the distribution of x_{ij} under \mathcal{H}_1 is exponential like the one under \mathcal{H}_0 , unlike [41] where the primary user signal is assumed to be deterministic, we can also use the above approach to derive analytical expressions for the local probability of detection, the ASN under \mathcal{H}_1 , and the censoring rate.

Denoting E_n to be the event where $a_i < \zeta_{ij} < b_i$, $i = 1, \dots, n-1$ and $\zeta_{nj} \geq b_n$, (30) becomes [41],

$$P_{fj} = \sum_{n=1}^N Pr(E_n | \mathcal{H}_0). \tag{45}$$

where the analytical expression for $Pr(E_n | \mathcal{H}_0)$ is derived in Appendix A.

Similarly for the local probability of detection, we have

$$P_{dj} = \sum_{n=1}^N Pr(E_n | \mathcal{H}_1), \tag{46}$$

where the analytical expression for $Pr(E_n | \mathcal{H}_0)$ is derived in Appendix B.

Defining $R_{nj} = \{\zeta_{ij} | \zeta_{ij} \in (a_i, b_i), i = 1, \dots, n\}$, $Pr(R_{nj} | \mathcal{H}_0)$ and $Pr(R_{nj} | \mathcal{H}_1)$ are obtained as follows

$$Pr(R_{nj} | \mathcal{H}_0) = \frac{1}{2^n} J_{a_n, b_n}^{(n)}(1/2), \quad n = 1, \dots, N, \tag{47}$$

$$Pr(R_{nj} | \mathcal{H}_1) = \frac{1}{[2(1 + \gamma_j)]^n} J_{a_n, b_n}^{(n)}(1/2(1 + \gamma_j)), \quad n = 1, \dots, N, \tag{48}$$

where $J_{a_n, b_n}^{(n)}(\theta)$ is presented in Appendix C and (36) and (37) become

$$E(N_j|\mathcal{H}_0) = \sum_{n=1}^{N-1} n(Pr(R_{n-1j}|\mathcal{H}_0) - Pr(R_{nj}|\mathcal{H}_0)) + NPr(R_{N-1j}|\mathcal{H}_0) = 1 + \sum_{n=1}^{N-1} Pr(R_{nj}|\mathcal{H}_0), \quad (49)$$

$$E(N_j|\mathcal{H}_1) = \sum_{n=1}^N n(Pr(R_{n-1j}|\mathcal{H}_1) - Pr(R_{nj}|\mathcal{H}_1)) + NPr(R_{N-1j}|\mathcal{H}_1) = 1 + \sum_{n=1}^{N-1} Pr(R_{nj}|\mathcal{H}_1). \quad (50)$$

With (49) and (50), we can calculate (35). This way, (33) and (34) can be derived as follows

$$\delta_{0j} = Pr(R_{Nj}|\mathcal{H}_0) = \frac{1}{2^N} J_{a_N, b_N}^{(N)}(1/2), \quad (51)$$

$$\delta_{1j} = Pr(R_{Nj}|\mathcal{H}_1) = \frac{1}{[2(1 + \gamma_j)]^N} J_{a_N, b_N}^{(N)}(1/2(1 + \gamma_j)). \quad (52)$$

It is easy to see that with (45), (46), (51), (52), (49) and (50), the problem (39) is not convex. Therefore, the standard systematic optimization algorithms do not give the global optimum for \bar{a} and \bar{b} . However, as is shown in the following lines, \bar{a} and \bar{b} are bounded and therefore, a two-dimensional exhaustive search is possible to find the global optimum. First of all, we have $a < 0$ and $\bar{a} < 0$. On the other hand, if \bar{a} has to play a role in the sensing system, at least one a_N should be positive, i.e., $a_N = \bar{a} + N\Delta \geq 0$ which gives $\bar{a} \geq -N\Delta$. Hence, we obtain $-N\Delta \leq \bar{a} < 0$. Furthermore, defining $Q_F^{cs} = \mathcal{F}(\bar{a}, \bar{b})$ and $Q_D^{cs} = \mathcal{G}(\bar{a}, \bar{b})$, for a given \bar{a} , it is easy to show that $\mathcal{G}^{-1}(\bar{a}, \beta) \leq \bar{b} \leq \mathcal{F}^{-1}(\bar{a}, \alpha)$.

Before introducing the suboptimal problem, the following theorem is presented.

Theorem 2. For a given local probability of detection and false alarm (P_d and P_f) and N , the censoring rate of the optimal censored truncated sequential sensing is less than the one of the censoring scheme.

Proof. Assume the P_f and P_d are the respective given local probability of false alarm and detection. Denoting ρ^c as the censoring rate for the optimal censoring scheme, we obtain $1 - \rho^c = \pi_0 P_f + \pi_1 P_d$, and denoting ρ^{cs} as the censoring rate for the optimal censored truncated sequential sensing, we obtain $1 - \rho^{cs} = \pi_0(P_f + \mathcal{L}_0(\bar{a}, \bar{b})) + \pi_1(P_d + \mathcal{L}_1(\bar{a}, \bar{b}))$. Note that $\mathcal{L}_k(\bar{a}, \bar{b})$, $k = 0, 1$, represents the probability that $\zeta_n \leq a_n$, $n = 1, \dots, N$ under \mathcal{H}_k which is non-negative. Hence, we can conclude that $1 - \rho^{cs} \geq 1 - \rho^c$ and thus $\rho^c \geq \rho^{cs}$. \square

Proposition 1. One corollary of Theorem 2 is that although the optimal solution of (12) or (23) for a specific N , i.e., $P_d = 1 - (1 - \beta)^{1/M}$ and $P_f = H^{-1}(\beta)$, is in the feasible set of (39) for a resulting ASN less than N , but it does not necessarily guarantee that the resulting energy consumption per sensor of the censored truncated sequential sensing approach is less than the one of the censoring scheme, particularly when the transmission cost is extremely higher than the sensing cost.

To solve (39), the problem is highly complex in terms of computation, and thus a two-dimensional exhaustive search is not always a good solution. Therefore, in order to reach a good solution in a reasonable time, we set $a < -N\Delta$ in order to obtain $a_1 = \dots = a_N = 0$. This way, we can relax one of the arguments of (39) and only solve the following suboptimal problem

$$\begin{aligned} \min_{\bar{b}} \max_j C_j \\ \text{s.t. } Q_F^{cs} \leq \alpha, \quad Q_D^{cs} \geq \beta. \end{aligned} \quad (53)$$

Note that unlike Section II, here the zero lower threshold is not necessarily optimal. The reason is that although the maximum censoring rate is achieved with the lowest \bar{a} , the minimum ASN is achieved with the highest \bar{a} , thus there is an inherent trade-off between a high censoring rate and a low ASN and a zero a_i is not necessarily the optimal solution. Since the analytical expressions provided earlier are very complex, we now try to provide a new set of analytical expressions for different parameters based on the fact that $a_1 \leq \dots \leq a_N \leq 0$.

To find the analytical expression for P_{fj} , we need to derive $A(n)$ for the new paradigm as follows

$$A(n) = \int \dots \int_{\Gamma_n} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{n-1j}\}} d\zeta_{1j} \dots d\zeta_{n-1j}. \quad (54)$$

Since $0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{n-1j}$ and $a_1 = \dots = a_N \leq 0$, the lower bound for each integral is ζ_{i-1} and the upper bound is b_i , where $i = 1, \dots, n-1$. Thus we obtain

$$A(n) = \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j}, \quad (55)$$

which according to [40] is

$$A(n) = \frac{b_1 b_n^{n-2}}{(n-1)!}, \quad n = 1, \dots, N. \quad (56)$$

Hence, we have

$$P_{fj} = \sum_{n=1}^N p_n A(n), \quad (57)$$

and $p_n = \frac{e^{-b_n/2}}{2^{n-1}}$. Similarly, for P_{dj} , we obtain

$$\begin{aligned} B(n) &= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\ &= \frac{b_1 b_n^{n-2}}{(n-1)!}, \quad n = 1, \dots, N, \end{aligned} \quad (58)$$

and thus,

$$P_{dj} = \sum_{n=1}^N q_n B(n), \quad (59)$$

where $q_n = \frac{e^{-b_n/2(1+\gamma_j)}}{[2(1+\gamma_j)]^{n-1}}$. Furthermore, we note that for $a_1 \leq \dots \leq a_N \leq 0$, $A(n) = B(n) = \frac{b_1 b_n^{n-2}}{(n-1)!}$, $n = 1, \dots, N$.

The analytical expression for $Pr(R_{nj}|\mathcal{H}_0)$ which is derived in Appendix D is

$$Pr(R_{nj}|\mathcal{H}_0) = 1 - \sum_{i=1}^n p_i A(i), \quad (60)$$

and for $Pr(R_{nj}|\mathcal{H}_1)$, according to Appendix E, we obtain

$$Pr(R_{nj}|\mathcal{H}_1) = 1 - \sum_{i=1}^n q_i A(i). \quad (61)$$

Putting (60) and (61) in (49) and (50), we obtain

$$E(N_j|\mathcal{H}_0) = 1 + \sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^n p_i A(i) \right\}, \quad (62)$$

$$E(N_j|\mathcal{H}_1) = 1 + \sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^n q_i A(i) \right\}, \quad (63)$$

and inserting (62) and (63) in (35), we obtain

$$\bar{N}_j = \pi_0 \left(1 + \sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^n p_i A(i) \right\} \right) + \pi_1 \left(1 + \sum_{n=1}^{N-1} \left\{ 1 - \sum_{i=1}^n q_i A(i) \right\} \right). \quad (64)$$

Finally, from (60) and (61), the censoring rate can be easily obtained as

$$\rho_j = \pi_0 \left(1 - \sum_{i=1}^N p_i A(i) \right) + \pi_1 \left(1 - \sum_{i=1}^N q_i A(i) \right). \quad (65)$$

Having the analytical expressions for (53), we can easily find the optimal maximum energy consumption per sensor by a line search over \bar{b} . Similar to the censoring problem formulation, here the sensing threshold is also bounded by $Q_F^{cs-1}(\alpha) \leq \bar{b} \leq Q_D^{cs-1}(\beta)$. As we will see in Section IV, censored truncated sequential sensing performs better than censored spectrum sensing in terms of energy efficiency in general. However the following theorem shows that as $\pi_0 \rightarrow 0$, censored truncated sequential sensing always outperforms the censoring scheme in case the SNRs are assumed to be the same.

Theorem 3. Censored truncated sequential spectrum sensing always outperforms the censoring scheme as $\pi_0 \rightarrow 0$ when $\gamma_1 = \dots = \gamma_M$.

Proof. One feasible solution of (53) is obtained by solving

$$\begin{aligned} \min_{\bar{b}} \max_j (1 - \rho_j) \\ \text{s.t. } P_f \leq 1 - (1 - \alpha)^{1/M}, \quad 1 - \prod_{i=1}^M (1 - P_{dj}) \geq 1 - (1 - P_d)^M \geq \beta, \end{aligned} \quad (66)$$

where P_d is the probability of detection of $\gamma_{min} = \min \{\gamma_1, \dots, \gamma_M\}$. Note that $\rho_j = \pi_0 \delta_{0j} + \pi_1 \delta_{1j}$ with $\delta_{0j} = 1 - P_f$ and $\delta_{1j} = 1 - P_{dj}$. Further considering the fact that P_f is an one to one function of \bar{b} we can rewrite (66) as follows,

$$\begin{aligned} \min_{P_f} \pi_0 P_f + \pi_1 P_d \\ \text{s.t. } P_f \leq 1 - (1 - \alpha)^{1/M}, \quad P_d \leq 1 - (1 - \beta)^{1/M} \end{aligned} \quad (67)$$

Since P_d is a monotonically increasing function of P_f such as censoring problem formulation, the optimal solution of (67) is $P_f = P_d^{-1}(1 - (1 - \beta)^{1/M})$ and $P_d = 1 - (1 - \beta)^{1/M}$. We can see that the optimal probability of detection for (67) is the same as the one in (23) in case the SNRs are assumed to be the same. However, it is not known if the optimal P_f is less than or equal to the one in (23). Note that, although considering \bar{a} as an argument for optimization as in (39), the optimal solution of (12) is in the feasible set of (39) but this is not necessarily the case for (53). Further, as Proposition 1 shows, for an optimal probability of detection for the censoring scheme, the censoring rate of sequential censoring is not bigger than the one for censoring scheme. Thus, it is seen that as $\pi_0 \rightarrow 0$, censored truncated sequential sensing performs at least the same as censored spectrum sensing. \square

IV. NUMERICAL RESULTS

A network of $M=5$ cognitive radios is considered for the simulations. For the sake of simplicity, it is assumed that all the sensors experience the same SNR. The cost of sensing per sample is $C_{sj} = 1$ and $C_{tj} = 10$. Further, the

probability of false alarm constraint $\alpha = 0.1$. In Fig. 2a the maximum energy consumption per sensor is optimized for $\gamma = 0\text{dB}$, $0.1 \leq \beta < 1$, and $\pi_0 = 0.2, 0.8$, and it is compared with the reference energy consumption where only censoring is employed by the cognitive radios. As we can see, the proposed censored truncated sequential scheme reduces the maximum energy consumption per sensor for both low and high π_0 as well as over the whole range of the detection probability constraint. Further, it is shown that the censored sequential scheme gives a higher energy efficiency than its censoring counterpart, particularly at high probability of detections. It is also shown that as π_0 increases, the maximum energy consumption per sensor decreases mainly due to a higher censoring rate.

Fig. 2b shows the optimal censoring rate versus β for the same scenario. Clearly, it is shown that the optimal censoring rate for higher π_0 is higher and further it is shown that the optimal censoring rate is slightly higher for censoring than for censored sequential sensing.

The optimal ASN versus β for the scenario of Fig. 2a is shown in Fig. 2c. We can see that as π_0 increases the optimal ASN also increases which is expected due to the smaller probability of primary user appearance. Further, if the probability of detection increases, the ASN decreases, because the threshold \bar{b} is lower for the higher detection rates and thus, cognitive radios sooner reach a decision.

Fig. 3a depicts the optimal maximum energy consumption per sensor versus the number of cognitive radios. Similar to the last scenario, the SNR is assumed to be 0 dB, $N = 10$, $C_s = 1$ and $C_t = 10$. Furthermore, the probability of false alarm and detection constraints are assumed to be $\alpha = 0.1$ and $\beta = 0.9$ as it is determined by the IEEE 802.15.4 standard for cognitive radios [8]. It is shown for both high and low values of π_0 that censored sequential sensing outperforms the censoring scheme. Looking at Fig. 3b and Fig. 3c, where the respective optimal censoring rate and optimal ASN are shown versus the number of cognitive radios, we can deduce that the lower ASN is playing a key role in lower energy consumption of the censored sequential sensing. Fig. 3a also shows that as the number of cooperating cognitive radios increases, the optimal maximum energy consumption per sensor decreases and saturates, while as is shown in Fig. 3b and Fig. 3c, the optimal censoring rate and optimal ASN increase. This way, the energy consumption tends to increase as a result of ASN growth and on the other hand inclines to decrease due to the censoring rate growth and that is the reason for saturation after a number of cognitive radios. Furthermore, as for Fig. 2c, the optimal ASN is lower for the low values of π_0 as a result of a higher probability of primary user presence and thus a faster decision making.

Figures 4a, 4b and 4c consider a scenario where $M = 5$, $N = 30$, $C_{sj} = 1$, $C_{tj} = 10$, $\alpha = 0.1$, $\beta = 0.9$ and π_0 can take a value of 0.2 or 0.8. The performance of the system versus SNR is analyzed in this scenario. The maximum energy consumption per sensor is depicted in Fig. 4a. As for the two earlier scenarios, censored sequential sensing gives a higher energy efficiency compared to censoring. While the optimal energy variation for the censoring scheme is almost the same for all the considered SNRs, the censored sequential scheme's energy consumption per sensor reduces significantly as the SNR increases. The reason is that as the SNR increases, the optimal ASN dramatically decreases (almost 50% for $\gamma = 2\text{ dB}$ and $\pi_0 = 0.2$).

Unlike the earlier scenarios, in Fig. 4a the optimal maximum energy per sensor for censored sequential sensing is lower for almost the whole SNR range (except for $\gamma = -4\text{ dB}$) for $\pi_0 = 0.2$. The reason is that in this scenario

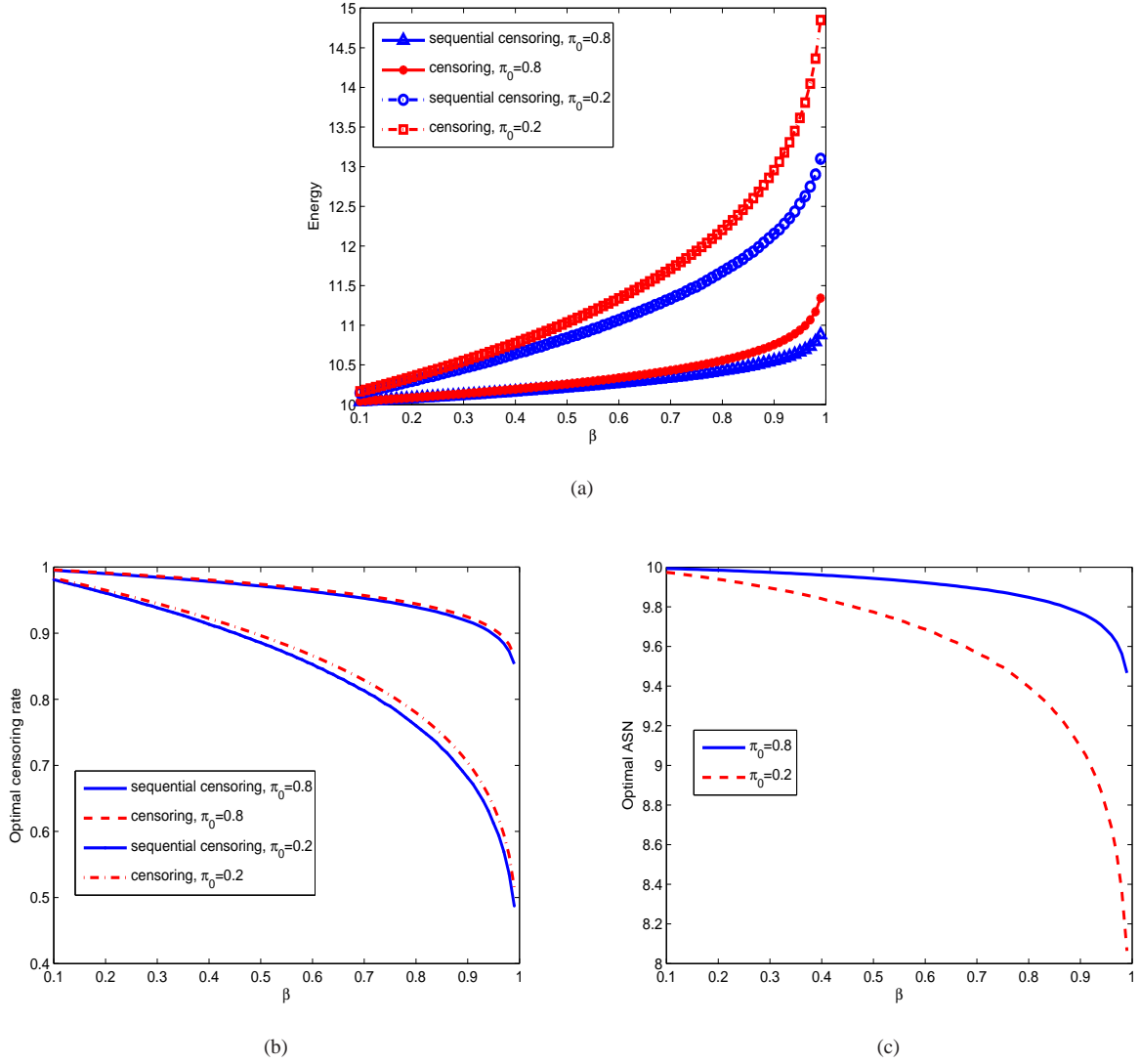


Fig. 2: a) Optimal maximum energy consumption per sensor versus β , b) Optimal censoring rate versus β , c) Optimal ASN versus β

the number of samples is assumed to be $N = 30$ and thus, the ASN which manages the total sensing cost becomes more important compared to the censoring rate that controls the transmission energy at higher SNRs. Since the optimal ASN for $\pi_0 = 0.2$ is much lower than its value for $\pi_0 = 0.8$, the energy consumption per sensor becomes lower, although the censoring rate for $\pi_0 = 0.8$ is higher. Furthermore, it is shown that as the number of samples increases, the censored sequential scheme gives a much lower energy consumption than its censoring counterpart.

The number of samples or truncation point N in censoring or censored sequential sensing is a key parameter. Although, it is not considered as an argument for optimization, it is important to discover its effect on the energy efficiency of the system. Figures 5, 6a, 7a, and 7b depict the maximum energy consumption per sensor versus N

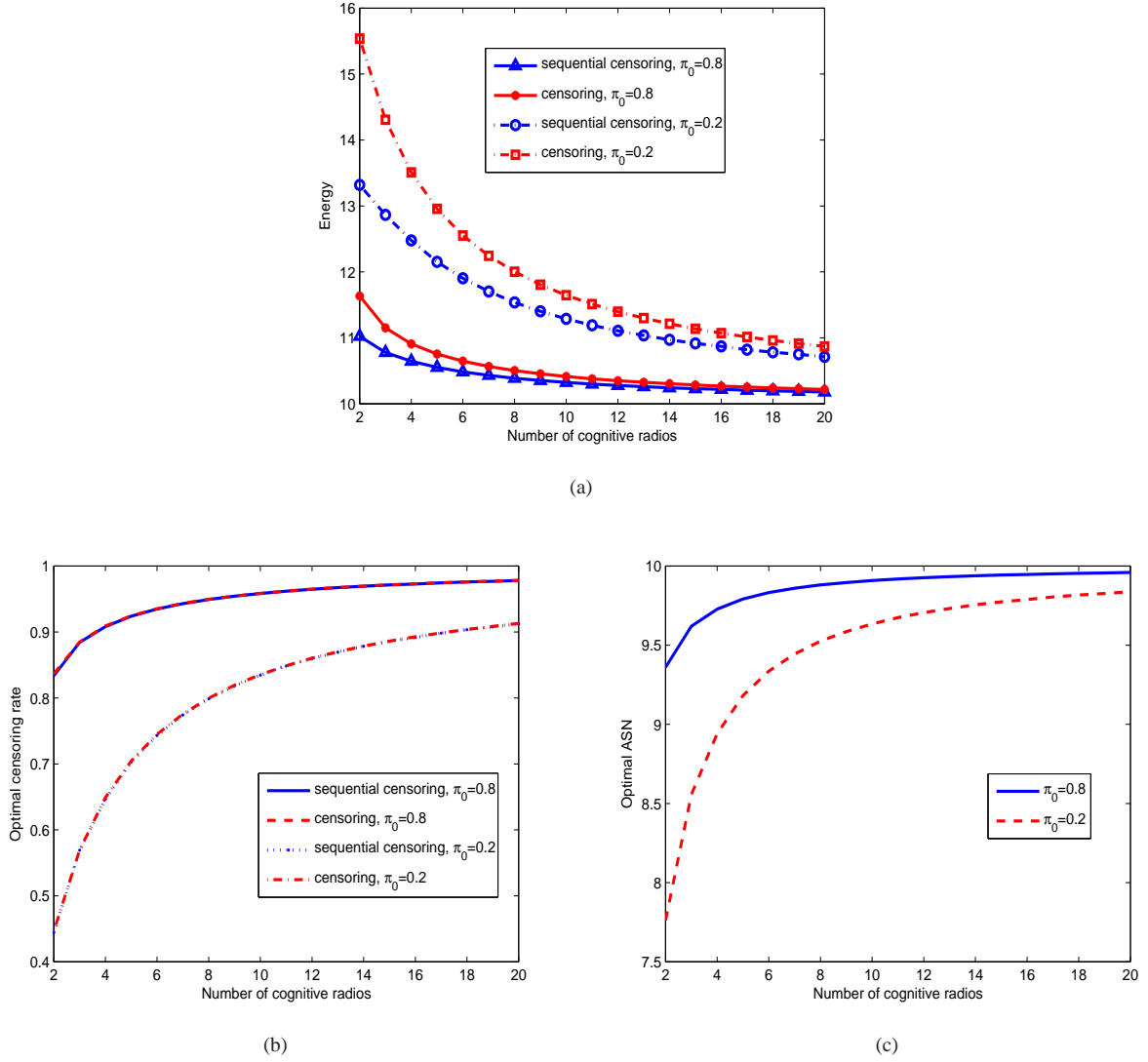


Fig. 3: a) Optimal maximum energy consumption per sensor versus number of cognitive radios, b) Optimal censoring rate versus number of cognitive radios, c) Optimal ASN versus number of cognitive radios

for respectively $C_t = 10$, $C_t = 100$, and $C_t = 1000$ while $C_s = 1$. Note that the assumptions of $C_t/C_s = 100$ or $C_t/C_s = 1000$ are not really valid for low-power radios where the transmission range and power is limited and most power is reserved for sensing (for example as for ZigBee in [23]). However, theoretically it is valuable to discuss cases where the transmission cost is much higher than the sensing cost. The system parameters for Figures 5, 6a, 6b, 6c, 7a and 7b are $M = 5$, $\gamma = 0$ dB, $\alpha = 0.1$, $\beta = 0.9$ and π_0 can take a value of 0.2 or 0.8.

It is shown in Fig. 5 that the optimal energy consumption per sensor linearly increases with N for a $C_t/C_s = 10$. Such an assumption accounts for an upper bound on C_t/C_s for most of the low-power communications standards. The censored sequential sensing scheme is shown to be more efficient in terms of energy consumption than the

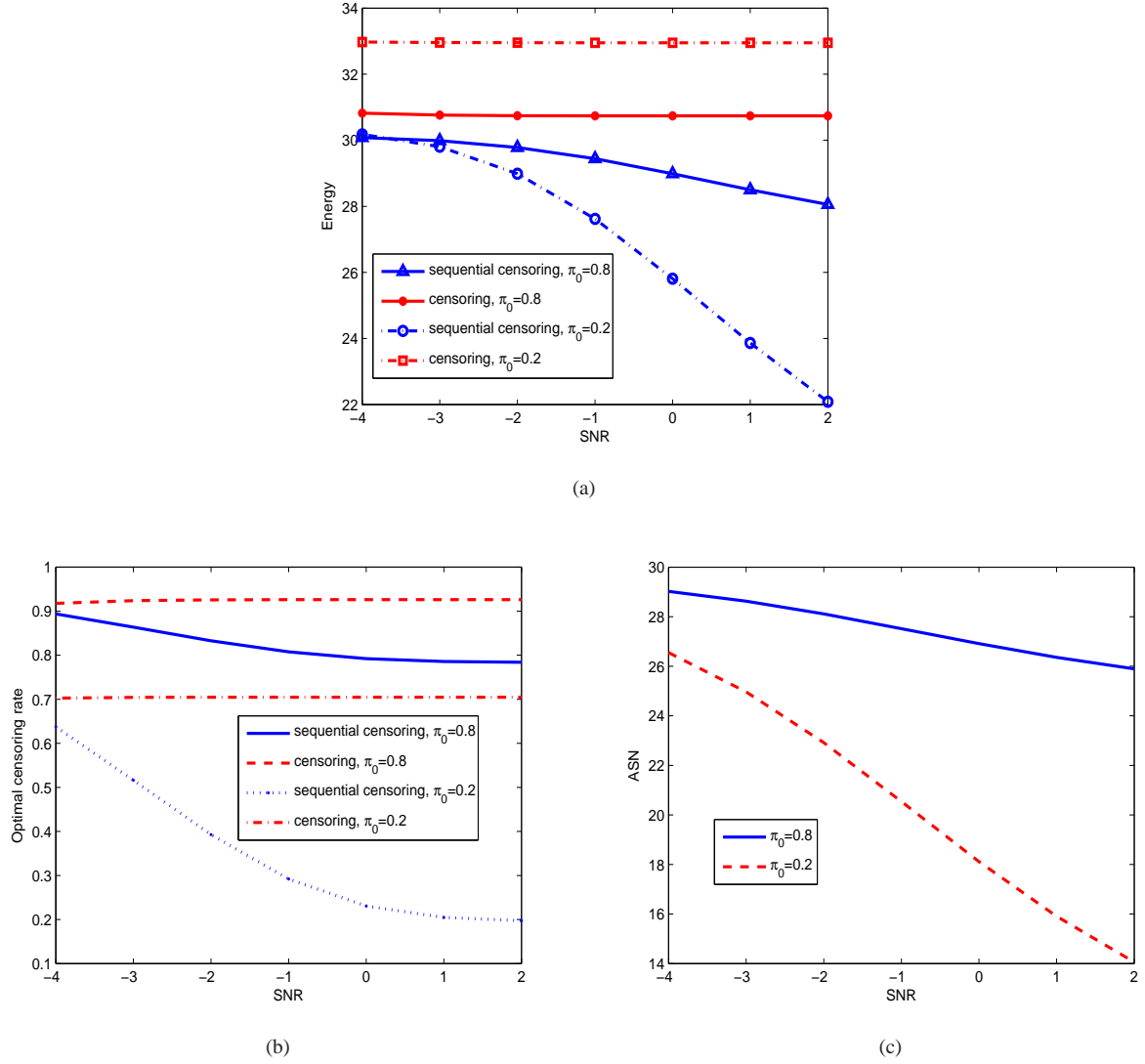


Fig. 4: a) Optimal maximum energy consumption per sensor versus SNR, b) Optimal censoring rate versus SNR, c) Optimal ASN versus SNR

censoring scheme. If the number of samples is considered as an argument for (12) and (53), then the minimum N that satisfies the constraint attains the optimal solution when $C_t/C_s \leq 10$. The same trend is shown in Fig. 6a where $C_t/C_s = 100$.

The effect of N is discussed in more detail for $C_t = 100$ where the optimal censoring rate and ASN for Fig. 6a is shown in Fig. 6b and Fig. 6c, respectively. The large difference between the maximum energy consumption per sensor for $\pi_0 = 0.2$ and $\pi_0 = 0.8$ is a result of a larger difference between the optimal censoring rate for $\pi_0 = 0.2$ and $\pi_0 = 0.8$.

The validity of Theorem 3 is illustrated in Fig. 7a and Fig. 7b where an extremely high transmission versus sensing

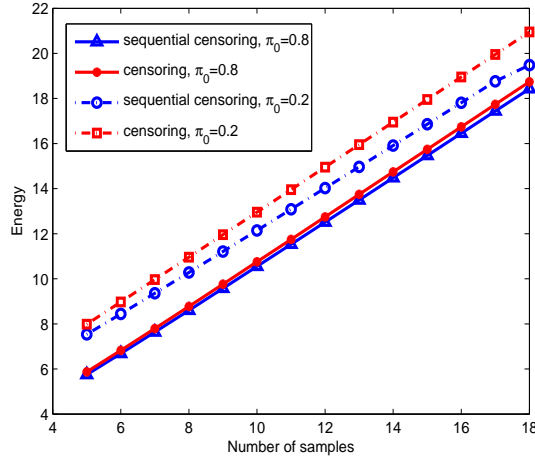


Fig. 5: Optimal maximum energy consumption per sensor versus number of samples

cost ($C_t/C_s = 1000$) is assumed for numerical results. It is depicted that the maximum energy consumption per sensor is not necessarily a linear increasing function of N but is more like a convex function. We can see that the optimal N for both censored sequential sensing and censoring is the same for both $\pi_0 = 0.2$ and $\pi_0 = 0.8$ and further, considering N as an argument for optimization, sequential censoring outperforms censoring. However, as is shown for the higher value of π_0 where the probability of primary user absence is high, for low values of N (less than 9 in Fig. 7b), the censoring formulation performs slightly better than the sequential censoring scheme for an extremely high transmission cost, due to a slightly higher optimal censoring rate. Nevertheless, as N increases and so the total sensing energy becomes larger, sequential censoring again outperforms censoring. Further, as is shown in Fig. 7a, as π_0 becomes low, sequential censoring always outperforms censoring which verifies our statement in Theorem 2.

Based on the analyses, it can be deduced that the optimal N is the minimum N that satisfies the probability of false alarm and detection constraints when the transmission cost is not much higher than the sensing one. However, when the transmission energy is much higher than the sensing energy, the optimal N is not necessarily the minimum feasible N .

Figures 8a and 8b compare the performance of the single threshold censored truncated sequential scheme with the one assuming two thresholds, i.e. \bar{a} and \bar{b} . In these figures, $M = 5$, $N = 10$, $\gamma = 0$ dB, $C_t = 10$, $\pi_0 = 0.2, 0.8$, and $\alpha = 0.1$, while β changes from 0.1 to 0.99. The sensing cost, C_s in Fig. 8a is assumed 1, while in Fig. 8b it is 3. It is shown that as the sensing cost increases, the energy efficiency of the double threshold scheme also increases compared to the one of the single threshold scheme, particularly when π_0 is high. The reason is that when π_0 is high, a much lower ASN can be achieved by the double threshold scheme compared to the single threshold one. This gain in performance comes at the cost of a high computational complexity. However, it seems that as the sensing cost and π_0 increases, the double threshold scheme becomes more attractive.

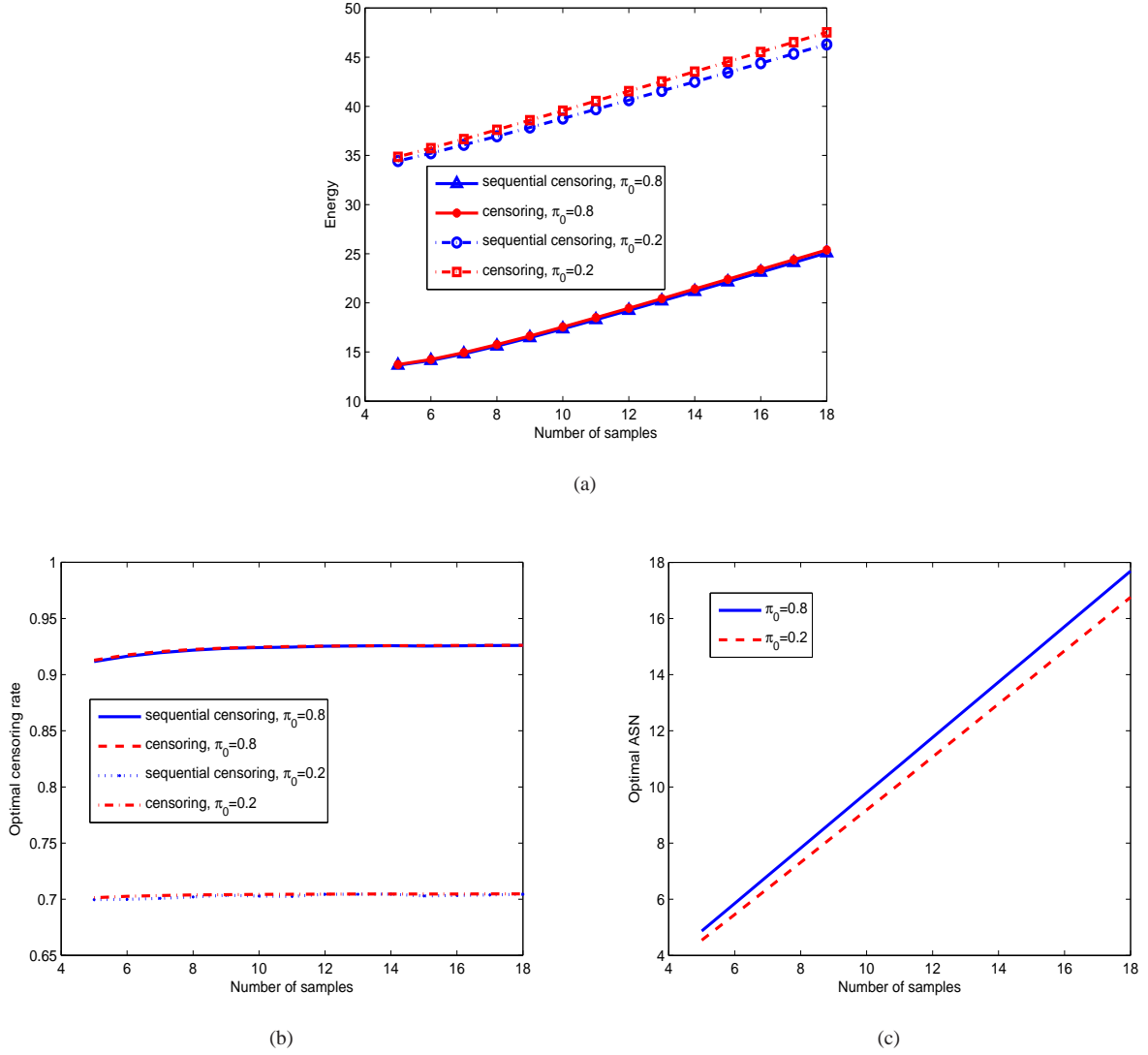
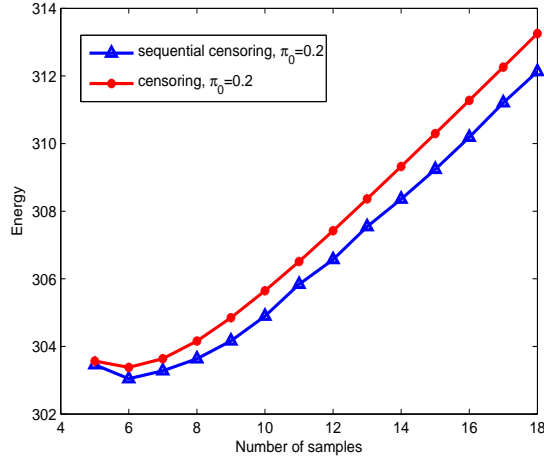


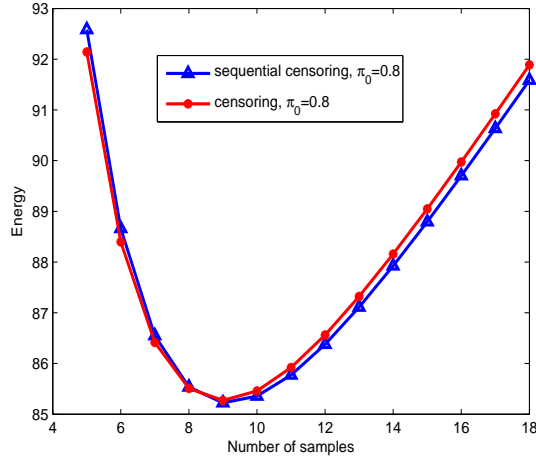
Fig. 6: a) Optimal maximum energy consumption per sensor versus number of samples, b) Optimal censoring rate versus number of samples, c) Optimal ASN versus number of samples

V. SUMMARY AND CONCLUSIONS

We presented two energy efficient techniques for a cognitive sensor network. First, a censoring scheme has been discussed where each sensor employs a censoring policy to reduce the energy consumption. Then a censored truncated sequential approach has been proposed based on the combination of censoring and sequential sensing policies. We defined our problem as the minimization of the maximum energy consumption per sensor subject to a global probability of false alarm and detection constraint. The optimal lower threshold is shown to be zero for the censoring scheme. Further, an explicit expression was given to find the optimal solution. We have further derived the analytical expressions for the underlying parameters in the censored sequential scheme and have shown



(a)

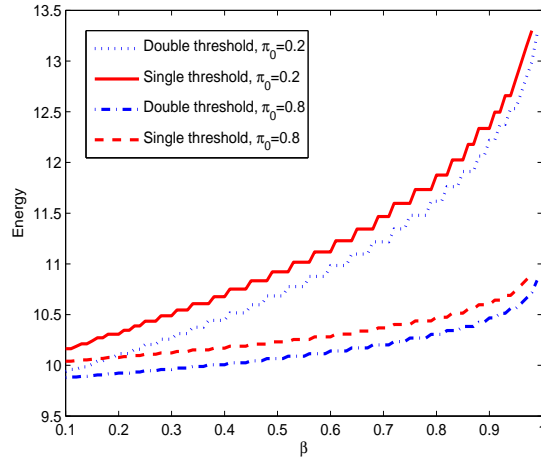


(b)

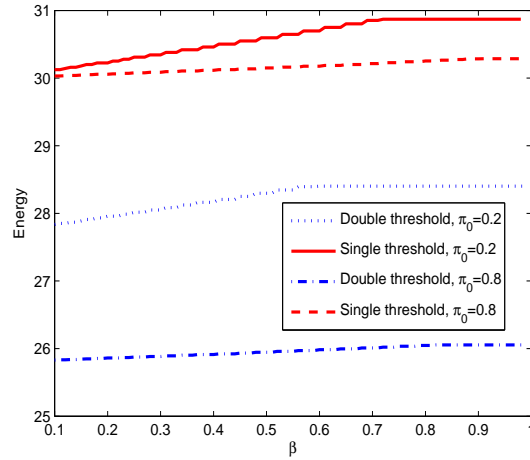
Fig. 7: Optimal maximum energy consumption per sensor versus number of samples, a) $\pi_0 = 0.2$, b) $\pi_0 = 0.8$

that although the problem is not convex, a bounded two-dimensional search is possible. Further, we relaxed the lower threshold to obtain a line search problem in order to reduce the computational complexity. The analytical expressions under the lower threshold relaxation have been extracted and it has been proved that for a given local probability of false alarm and detection, the associated censoring rate is higher for the optimal censoring scheme than for censored sequential sensing. Furthermore, as π_0 tends to zero, the censored sequential approach has been shown to be always better than the censoring scheme.

Different scenarios regarding transmission and sensing cost as well as SNR, number of cognitive radios, number of samples and probability of detection constraints were simulated for low and high values of π_0 . It has been shown that under the practical assumption of low-power radios, sequential censoring outperforms censoring. However



(a)



(b)

Fig. 8: Optimal maximum energy consumption per sensor versus probability of detection constraint, β , a) $C_s = 1$, b) $C_s = 3$

as the transmission cost becomes much higher than the sensing cost, for a small number of sample observations and high π_0 's, the censoring scheme might slightly perform better than the sequential scheme in terms of energy efficiency. Further, it has been shown that the optimal ASN for low values of π_0 is lower than for high values and the same trend is also valid for the optimal censoring rate. We conclude that for high values of the sensing cost, despite its high computational complexity, the double threshold scheme becomes more attractive.

Note that a systematic solution for the censored sequential problem formulation was not given in this paper, and thus it is valuable to investigate a better algorithm to solve the problem. We also did not consider a combination of the proposed scheme with sleeping as in [23]. Further energy savings can be obtained by incorporating a sleeping

policy. The numerical results have shown that under extreme circumstances, considering the number of observation samples or truncation point is worthwhile. Such an optimization problem is subject of further work. Furthermore, our analysis was based on the OR rule, and thus extensions to other hard fusion rules could be interesting. Quickest change detection is another type of sequential detection where an abrupt change is to be detected by the sensor system [46]. This approach is useful when a primary user suddenly appears in a cognitive radio system and to design an efficient out-of-band sensing. Energy optimization in such a system is also subject of further research.

APPENDIX A

DERIVATION OF $Pr(E_n|\mathcal{H}_0)$

Introducing $\Gamma_n = \{a_i < \zeta_{ij} < b_i, i = 1, \dots, n-1\}$ and $p_n = \frac{1}{2^{n-1}}e^{-b_n/2}$, we can write

$$\begin{aligned} Pr(E_n|\mathcal{H}_0) &= \int_{\Gamma_n} \dots \int_{b_n}^{\infty} \frac{1}{2^n} e^{-\zeta_{nj}/2} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{nj}\}} d\zeta_{1j} \dots d\zeta_{nj} \\ &= p_n \int_{\Gamma_n} \dots \int I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{n-1j}\}} d\zeta_{1j} \dots d\zeta_{n-1j}. \end{aligned} \quad (68)$$

Denoting $A(n) = \int_{\Gamma_n} \dots \int I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{n-1j}\}} d\zeta_{1j} \dots d\zeta_{n-1j}$, we obtain

$$A(n) = \begin{cases} \frac{b_1 b_n^{n-2}}{(n-1)!}, & n = 1, \dots, p+1 \\ [f_{a_0^{n-1}}^{(n-1)}(b_{n-1}) - I_{\{n \geq 3\}} \sum_{i=0}^{n-3} \frac{(b_{n-1}-b_{i+1})^{n-i-1}}{(n-i-1)!} 2^i e^{\frac{b_{i+1}}{2}} Pr(E_{i+1}|\mathcal{H}_0)], & n = p+2, \dots, q+1, \\ [f_{a_0^{n-1}}^{(n-1)}(b_{n-1}) - \sum_{i=0}^n f_{\psi_{i,a_{n-1}}^{n-1}}^{(n-1-i)}(b_{n-1}) 2^i e^{\frac{b_{i+1}}{2}} Pr(E_{i+1}|\mathcal{H}_0)], & n = q+2, \dots, N \end{cases} \quad (69)$$

where $a_0^{n-1} = [a_0, \dots, a_{n-1}]$. Denoting q to be the smallest integer for which $a_q \leq b_1 < b_q$, and c and d to be two non-negative real numbers satisfying $0 \leq c < d$, $a_{n-1} \leq c \leq b_n$ and $a_n \leq d$, $\eta_0 = 0$, $\boldsymbol{\eta}_k = [\eta_1, \dots, \eta_k]$, $0 \leq \eta_1 \leq \dots \leq \eta_k$, the functions $f_{\boldsymbol{\eta}_k}^{(k)}(\zeta)$ and the vector $\boldsymbol{\psi}_{i,c}^n$ in (69) are as follows

$$\begin{aligned} f_{\boldsymbol{\eta}_k}^{(k)}(\zeta) &= \sum_{i=0}^{k-1} \frac{f_i^{(k)}(\zeta - \eta_{i+1})^{k-i}}{(k-i)!} + f_k^{(k)} \\ f_i^{(k)} &= f_i^{(k-1)}, \quad i = 0, \dots, k-1, \quad k \geq 1, \quad f_k^{(k)} = - \sum_{i=0}^{k-1} \frac{f_i^{(k-1)}}{(k-i)!} (\eta_k - \eta_{i+1})^{k-i}, \quad f_0^{(0)} = 1, \end{aligned} \quad (70)$$

$$\boldsymbol{\psi}_{i,c}^n = \begin{cases} [\underbrace{b_{i+1}, \dots, b_{i+1}}_q, \underbrace{a_{q+i+1}, \dots, a_{n-1}, c}_{n-q-i}], & i \in [0, n-q-2] \\ [\underbrace{b_{i+1}, \dots, b_{i+1}, c}_{n-i}], & i \in [n-q-1, s-1] \\ b_{i+1} \mathbf{1}_{n-i}, & i \in [s, n-2] \end{cases}, \quad (71)$$

with s denoting the integer for which $b_s < c \leq b_{s+1}$ and $f_{\boldsymbol{\eta}_k}^{(0)}(\zeta) = 1$.

APPENDIX B

DERIVATION OF $Pr(E_n|\mathcal{H}_1)$

Introducing $q_n = \frac{1}{[2(1+\gamma_j)]^{n-1}} e^{-b_n/2(1+\gamma_j)}$, we can write

$$\begin{aligned} Pr(E_n|\mathcal{H}_1) &= \int \dots \int_{\Gamma_n} \int_{b_n}^{\infty} \frac{1}{[2(1+\gamma_j)]^n} e^{-\zeta_{nj}/2(1+\gamma_j)} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{nj}\}} d\zeta_{1j} \dots d\zeta_{nj} \\ &= q_n \int \dots \int_{\Gamma_n} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{n-1j}\}} d\zeta_{1j} \dots d\zeta_{n-1j}. \end{aligned} \quad (72)$$

Denoting $B(n) = \int \dots \int_{\Gamma_n} I_{\{0 \leq \zeta_{1j} \leq \zeta_{2j} \dots \leq \zeta_{n-1j}\}} d\zeta_{1j} \dots d\zeta_{n-1j}$, we obtain

$$B(n) = \begin{cases} \frac{b_1 b_n^{n-2}}{(n-1)!}, & n = 1, \dots, p+1 \\ [f_{\mathbf{a}_0^{n-1}}^{(n-1)}(b_{n-1}) - I_{\{n \geq 3\}} \sum_{i=0}^{n-3} \frac{(b_{n-1}-b_{i+1})^{n-i-1}}{(n-i-1)!} [2(1+\gamma_j)]^i e^{\frac{b_{i+1}}{2(1+\gamma_j)}} Pr(E_{i+1}|\mathcal{H}_1)], & n = p+2, \dots, q+1. \\ [f_{\mathbf{a}_0^{n-1}}^{(n-1)}(b_{n-1}) - \sum_{i=0}^{n-3} f_{\psi_{i, a_{n-1}}}^{(n-1-i)}(b_{n-1}) [2(1+\gamma_j)]^i e^{\frac{b_{i+1}}{2(1+\gamma_j)}} Pr(E_{i+1}|\mathcal{H}_1)], & n = q+2, \dots, N \end{cases} \quad (73)$$

APPENDIX C

 $J_{a_n, b_n}^{(n)}(\theta)$ ANALYTICAL EXPRESSION

Under $\theta > 0$, $n \geq 1$ and $0 \leq \zeta_{1j} \leq \dots \leq \zeta_{nj}$, $\zeta_{ij} \in (a_i, b_i)$, $i = 1, \dots, n$, the function $J_{a_n, b_n}^{(n)}(\theta)$ is defined as [41]

$$J_{a_n, b_n}^{(n)}(\theta) = \sum_{i=1}^n \theta^{-i} [f_{\mathbf{a}_0^{n-i}}^{(n-i)}(a_n) e^{-\theta a_n} - f_{\mathbf{a}_0^{n-i}}^{(n-i)}(b_n) e^{-\theta b_n}] - I_{\{n \geq 2\}} \sum_{k=0}^{n-2} g_{a_n, b_n}^{(k)}(\theta), \quad (74)$$

where [41]

$$g_{c,d}^{(k)} = \begin{cases} I^{(k)} [\theta^{k-n} e^{-\theta b_{k+1}} - \sum_{i=1}^{n-k} \theta^{-i} f_{b_{k+1} \mathbf{1}_{n-k-i}}^{(n-k-i)}(d) e^{-\theta d}], & c \leq b_1, \quad k \in [0, n-2] \\ I^{(k)} \sum_{i=1}^{n-k} \theta^{-i} [f_{\psi_{k,c}^{n-i}}^{(n-k-i)}(c) e^{-\theta c} - f_{\psi_{k,d}^{n-i}}^{(n-k-i)}(d) e^{-\theta d}], & c > b_1, \quad k \in [0, s-1], \\ I^{(k)} [\theta^{k-n} e^{-\theta b_{k+1}} - \sum_{i=1}^{n-k} \theta^{-i} f_{b_{k+1} \mathbf{1}_{n-k-i}}^{(n-k-i)}(d) e^{-\theta d}], & c > b_1, \quad k \in [s, n-2] \end{cases} \quad (75)$$

with $I^{(0)} = 1$ and

$$I^{(n)} = \begin{cases} f_{\mathbf{a}_0^n}^{(n)}(b_n) - I_{\{n \geq 2\}} \sum_{i=0}^{n-2} \frac{(b_n - b_{i+1})^{n-i}}{(n-i)!} I^{(i)}, & n \in [1, q] \\ f_{\mathbf{a}_0^n}^{(n)}(b_n) - \sum_{i=0}^{n-2} f_{\psi_{i, a_n}^{n-i}}^{(n-i)}(b_n) I^{(i)}, & n \in [q+1, \infty) \end{cases} \quad (76)$$

APPENDIX D

DERIVATION OF $Pr(R_{nj}|\mathcal{H}_0)$

The analytical expression for $Pr(R_{nj}|\mathcal{H}_0)$ is derived as

$$\begin{aligned}
Pr(R_{nj}|\mathcal{H}_0) &= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-1j}}^{b_n} \frac{1}{2^n} e^{-b_n/2} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{nj} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \left[\int_{\zeta_{n-1j}}^{b_n} \frac{1}{2^n} e^{-b_n/2} d\zeta_{nj} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{2^{n-1}} \left[e^{-\zeta_{n-1j}/2} - e^{-b_n/2} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{2^{n-1}} e^{-\zeta_{n-1j}/2} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&\quad - \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{2^{n-1}} e^{-b_n/2} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{2^{n-1}} e^{-\zeta_{n-1j}/2} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&\quad - \underbrace{\frac{1}{2^{n-1}} e^{-b_n/2}}_{p_n} \underbrace{\frac{b_1 b_n^{n-2}}{(n-1)!}}_{A(n)} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \left[\int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{2^{n-1}} e^{-\zeta_{n-1j}/2} d\zeta_{n-1j} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-2j} \\
&\quad - p_n A(n) \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-3j}}^{b_{n-2}} \frac{1}{2^{n-2}} \left[e^{-\zeta_{n-2j}/2} - e^{-b_{n-1}/2} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-2j} \\
&\quad - p_n A(n) \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-3j}}^{b_{n-2}} \frac{1}{2^{n-2}} e^{-\zeta_{n-2j}/2} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-2j} \\
&\quad - p_{n-1} A(n-1) - p_n A(n) \\
&= . \\
&= . \\
&= 1 - p_1 A(1) - p_2 A(2) - \dots - p_n A(n) \\
&= 1 - \sum_{i=1}^n p_i A(i). \tag{77}
\end{aligned}$$

APPENDIX E
DERIVATION OF $Pr(R_{nj}|\mathcal{H}_1)$

The analytical expression for $Pr(R_{nj}|\mathcal{H}_1)$ is derived as

$$\begin{aligned}
Pr(R_{nj}|\mathcal{H}_1) &= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-1j}}^{b_n} \frac{1}{[2(1+\gamma_j)]^n} e^{-b_n/2(1+\gamma_j)} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{nj} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \left[\int_{\zeta_{n-1j}}^{b_n} \frac{1}{[2(1+\gamma_j)]^n} e^{-b_n/2(1+\gamma_j)} d\zeta_{nj} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{[2(1+\gamma_j)]^{n-1}} \left[e^{-\zeta_{n-1j}/2(1+\gamma_j)} - e^{-b_n/2(1+\gamma_j)} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{[2(1+\gamma_j)]^{n-1}} e^{-\zeta_{n-1j}/2(1+\gamma_j)} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&\quad - \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{[2(1+\gamma_j)]^{n-1}} e^{-b_n/2(1+\gamma_j)} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{[2(1+\gamma_j)]^{n-1}} e^{-\zeta_{n-1j}/2(1+\gamma_j)} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-1j} \\
&\quad - \underbrace{\frac{1}{[2(1+\gamma_j)]^{n-1}}}_{q_n} \underbrace{\frac{b_1 b_n^{n-2}}{(n-1)!}}_{A(n)} \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \left[\int_{\zeta_{n-2j}}^{b_{n-1}} \frac{1}{[2(1+\gamma_j)]^{n-1}} e^{-\zeta_{n-1j}/2(1+\gamma_j)} d\zeta_{n-1j} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-2j} \\
&\quad - q_n A(n) \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-3j}}^{b_{n-2}} \frac{1}{[2(1+\gamma_j)]^{n-2}} \left[e^{-\zeta_{n-2j}/2(1+\gamma_j)} - e^{-b_{n-1}/2(1+\gamma_j)} \right] d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-2j} \\
&\quad - q_n A(n) \\
&= \int_{\zeta_{0j}}^{b_1} \int_{\zeta_{1j}}^{b_2} \dots \int_{\zeta_{n-3j}}^{b_{n-2}} \frac{1}{[2(1+\gamma_j)]^{n-2}} e^{-\zeta_{n-2j}/2(1+\gamma_j)} d\zeta_{1j} d\zeta_{2j} \dots d\zeta_{n-2j} \\
&\quad - q_{n-1} A(n-1) - q_n A(n) \\
&= . \\
&= . \\
&= 1 - q_1 A(1) - q_2 A(2) - \dots - q_n A(n) \\
&= 1 - \sum_{i=1}^n q_i A(i). \tag{78}
\end{aligned}$$

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